

Inventory model with inventory-level dependent demand rate, variable holding cost and shortages

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Abstract

Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales. Previous models incorporating stock-level dependent demand rate assume that the holding cost is constant for the entire inventory cycle. In this model we will discuss an inventory-level dependent demand rate, variable holding cost and shortages. The holding cost per unit of the item per unit time is assumed to be linear function of the quantity in storage. Procedures are developed for determining the optimal order quantity and the optimal cycle time.

Keywords: Inventory models, Inventory -dependent demand, Shortages

1. Introduction

As we know that in all traditional inventory models, the demand rate is assumed to be a given constant. There are various inventory models have been developed for dealing with varying and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an external parameter not influenced by the internal inventory policy. In real life, however, it is frequently observed that demand for a particular product can indeed be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

All those models that consider demand variation in response to inventory level assume that the holding cost is constant for the entire inventory cycle and shortages not allowed. This paper presents an inventory model with a linear inventory-level dependent demand rate and a variable holding cost and shortages. In this model, the holding cost is an increasing step function of the time spent in storage.

This structure is representative of many real-life situations in which distinctive unit holding cost occurs depends on the quantity kept in warehouse. This is particularly true in the storage of deteriorating and perishable items such as fruits in which quantity decreasing every time so the holding cost.

2. Problem definition and scope

The main objective of this paper is to determine the optimum (i.e. minimum cost) inventory policy for an inventory system with inventory-level dependent demand rate variable holding cost and shortages. Assuming the demand rate to be inventory-level dependent means the demand is higher for greater inventory levels. Assuming the holding cost per unit of the item per unit time to be time-dependent means the unit holding cost is higher for longer storage periods and holding cost considered to be zero when shortages occurs. The model that will be developed for the inventory system is based on allowing unit holding cost values to vary with different storage periods. Variable unit holding costs are considered in the model in determining the optimal inventory policy.

The holding cost per unit is assumed to increase only when the storage time exceeds specified discrete values. In other words, the holding cost per unit per unit time is an increasing step function of the storage time.

In this model we will consider Retroactive increase holding cost. In retroactive increase, the unit holding cost rate of the last storage period is applied to all storage periods. In this case holding cost considered to be zero when shortages occur.

2.1. Notation

$I(t)$ The inventory on-hand at time t

R Constant demand rate

N Number of distinct time periods with different holding cost rates

t Time

t_i End time of period i ,
where $i = 1, 2, 3, \dots, n$.

$t_0 = 0$, and $t_n = \infty$

A Ordering cost per order

h_i Holding cost of the item in period i

$h(t)$ Holding cost of an item at time t ,

$h(t) = h_i$ if $t_{i-1} \leq t \leq t_i$

T Cycle time

β Demand parameter indicating elasticity in relation to the inventory level

Q Quantity ordered

c Shortage cost per unit short

S Order level

2.2. Assumption and limitations

1. Replenishments rate is infinite.
2. Replenishment size is finite.
3. Lead time zero.
4. Every time we place the order of Q quantity units at the beginning of the cycle is such that the order level reaches to the prespecified value S and after fulfilling the backorder quantity $Q-S$.
5. Inventory level reaches to zero at time t_k and after this time shortages takes place
6. The demand rate R is linearly increasing function of the inventory level $I(t)$.
7. The holding cost is varying as an increasing step function of the quantity in storage.

8. Shortages, if any are backlogged and are satisfied when new lot arrived.
9. A single item is considered.
10. The demand rate R is linear function of the inventory level q which is expressed as

$$R(I(t)) = R + \beta I(t),$$

$$R > 0, 0 < \beta < 1, I(t) \geq 0.$$

2.3 Inventory Model

Our main objective is to minimize the TIC per unit time, which includes three components: The ordering cost, the holding cost and the shortage cost. Since one order is made per cycle, the ordering cost per unit time is given as follows

$$\text{Ordering cost per cycle} = \frac{A}{T}$$

As we have assume that the shortages occurs at time t_k we can write $I(t_k) = 0$.

Holding cost per cycle can be obtained as follows

$$\text{IHC} = \int_0^{t_k} h(t) I(t) dt \quad 2.3.1$$

Q quantity ordered and realized in stock and after fulfilling the back orders $Q-S$ the order level reaches to the order level S .

At the end of the cycle the inventory level reaches to $Q-S$.

The instantaneous states of $I(t)$ in the interval $(0, T)$ is given by

$$\frac{dI(t)}{dt} = \begin{cases} -(R + \beta I(t)); 0 \leq t < t_k \\ -R & ; t_k \leq t < T \end{cases} \quad 2.3.2$$

By solving the above D.E. we get

$$\frac{dI(t)}{(R + \beta I(t))} = -dt$$

Now by integrating both sides we get

$$\frac{1}{\beta} [\ln(R + \beta I(t)) - \ln(R + \beta I(0))] = -t$$

$$\ln\left(\frac{R + \beta I(t)}{R + \beta S}\right) = -\beta t$$

$$R + \beta I(t) = (R + \beta S)e^{-\beta t}$$

$$I(t) = \frac{1}{\beta} [(R + \beta S)e^{-\beta t} - R]; 0 \leq t < t_k \quad 2.3.3$$

Now from 2.3.2

$$\frac{dI(t)}{dt} = -R$$

On solving this we get

$$I(t) = -R(t - t_k); t_k \leq t \leq T \quad 2.3.4$$

$$I(t) = \begin{cases} \frac{1}{\beta} [(R + \beta S)e^{-\beta t} - R]; 0 \leq t < t_k \\ -R(t - t_k) & ; t_k \leq t \leq T \end{cases} \quad 2.3.5$$

Now from 2.3.5

$$I(T) = -R(T - t_k) = Q - S$$

$$\therefore T = t_k - \frac{Q - S}{R} \quad 2.3.6$$

$$\therefore Q = S - R(T - t_k) \quad 2.3.7$$

Since at $t = t_k$; $I(t) = 0$ using 2.3.5 we can write

$$\left(\frac{R}{\beta} + S\right) e^{-\beta t_k} - \frac{R}{\beta} = -R(0) = 0$$

$$\therefore e^{-\beta t_k} = \left(\frac{R}{\beta} + S\right) \frac{\beta}{R}$$

$$\therefore t_k = \frac{1}{\beta} \ln\left(\left(\frac{R}{\beta} + S\right) \frac{\beta}{R}\right) \quad 2.3.8$$

Cost of shortages

$$\begin{aligned} SC &= -c \int_{t_k}^T I(t) dt \\ &= -c \int_{t_k}^T R(t - t_k) dt \end{aligned}$$

$$\therefore SC = -cR \left[\frac{(T - t_k)^2}{2} \right] \quad 2.3.9$$

2.4 Retroactive Holding Cost

In this case holding cost of last storage period applies retroactively to all previous periods, thus h_k is the holding cost applied to all the previous periods because after the period t_{k-1} to t_k shortages takes place.

Holding Cost

$$\begin{aligned} H &= h_k \int_0^{t_k} I(t) dt \\ &= h_k \int_0^{t_k} \left[\left(\frac{R}{\beta} + S \right) e^{-\beta t} - \frac{R}{\beta} \right] dt \end{aligned}$$

$$\therefore H = h_k \left\{ \frac{1}{\beta} \left[\frac{R}{\beta} + S \right] (1 - e^{-\beta t}) - \frac{R}{\beta} t_k \right\} \quad 2.4.1$$

TIC = Ordering Cost + Shortage Cost + Holding Cost

Now put the values of SC and HC from 2.3.9 and 2.4.1 we get

$$\begin{aligned} &= \frac{A}{T} + \frac{-cR}{T} \left[\frac{(T - t_k)^2}{2} \right] \\ &\quad + h_k \left[\frac{1}{\beta} \left[\frac{R}{\beta} + S \right] (1 - e^{-\beta t}) - \frac{R}{\beta} t_k \right] \quad 2.4.2 \end{aligned}$$

Substitute the value of T from 2.3.6

$$TIC = \frac{R}{Rt_k - Q + S} \left[A - \frac{c(Q-S)^2}{2R} + h_k M \right] \quad 2.4.3$$

$$\text{Where } M = \frac{1}{\beta} \left(S + \frac{R}{\beta} \right) (1 - e^{-\beta t}) - \frac{R}{\beta} t_k$$

Now differentiating with respect to Q and equating to zero we get the following

$$\frac{\partial(TIC)}{\partial Q} = \frac{-c((Q-S)Rt_k - Q + S) + R \left(A - \frac{c(Q-S)^2}{2R} + h_k M \right)}{(Rt_k - Q + S)^2}$$

Now since $\frac{\partial(TIC)}{\partial Q} = 0$ we get

$$\begin{aligned} &c(Q-S)(Rt_k - Q + S) \\ &\quad + R \left(A - \frac{c(Q-S)^2}{2R} + h_k M \right) \\ &= 0 \end{aligned}$$

$$cR(Q-S)t_k - \frac{c(Q-S)^2}{2} = RA + RMh_k$$

$$\begin{aligned} \frac{c}{2} [-(Q-S)^2 + 2R(Q-S)t_k + R^2 t_k^2 \\ - R^2 t_k^2] = RA + RMh_k \end{aligned}$$

$$(Q-S + Rt_k)^2 = -\frac{2}{c} (RA + RMh_k) + R^2 t_k^2$$

$$Q = S - Rt_k + \sqrt{R^2 t_k^2 - \frac{2}{c} (RA + RMh_k)}$$

2.6 Conclusions and suggestions

A model has been presented of an inventory system with stock-dependent demand, in which the holding cost is a decreasing step function of the quantity in storage. In this model I have considered retroactive increase holding cost decrease based on the formulas developed, it can be concluded that both the optimal order quantity and the cycle time decrease when the holding cost increases. The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended to variable ordering costs, and non-instantaneous receipt of orders. The case of the decreasing holding cost considered in this paper applies rented storage facilities, where lower rent rates are normally obtained for longer-term leases.

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